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ENVIRONMENTAL QUALITY, THE MACROECONOMY, AND INTERGENERATIONAL DISTRIBUTION

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Environmental Quality, the Macroeconomy, and Intergenerational Distribution

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Abstract

The paper studies the dynamic allocation effects and intergenerational welfare consequences of environmental taxes. To this end, environmental externalities are introduced in a Blanchard-Yaari overlapping generations model of a small open economy. A rise in environmental taxes—taking into account pre-existing distortionary taxes and endogenous labor supply—is shown to yield an efficiency gain if agents care enough for the environment. The benefits are unevenly distributed across generations because agents are heterogeneous in their capital ownership. An accompanying debt policy can be designed—prescribing debt accumulation at impact and debt redemption in the new steady state—to ensure everybody gains to the same extent. With lump-sum recycling of environmental tax revenue, aggregate employment is unaffected in the short run, but falls in the long run. It raises environmental quality more in the long run than in the short run. Recycling revenue through a cut in labor taxes, however, is shown to yield a rise in employment in the short run, which disappears during transition. In the new steady state, environmental quality is higher at the expense of a lower level of employment.

JEL classification codes: D60, H23, H63, Q28.

Keywords: Yaari-Blanchard overlapping generations, environmental taxes, intergenerational distribution, public debt policy, double dividend hypothesis.

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1 Introduction

Many environmental problems have a cumulative character. Over time a stock of waste or pollution is built up, damaging the quality of our life-support system now and in the future. Current decisions on the use of the environment made by short-lived individuals therefore have long-lasting effects. Accordingly, welfare of both present and future generations is affected, that is, there is both an *intra*temporal and *inter*temporal external effect at work. Environmental policy should therefore pay attention to both the efficiency question of 'how much pollution to allow' and the intergenerational equity question of 'which generation pays for the environment and by how much.' Unfortunately, in much of the existing literature attention has been unduly focused on only the first aspect, and this paper is aimed to partially fill this void.

Early formal literature analyzing the environment-economy interaction—see Solow (1974) and Stiglitz (1974), and more recently Thavonen and Kuuluvainen (1991) and Van der Ploeg and Withagen (1991)—employs an infinitely-lived representative agent model, which makes it hard to study intergenerational issues. Indeed, to the extent that this fictional agent really constitutes a shortcut description of a dynasty of finitely-lived and altruistically linked generations, there is no intergenerational external effect to worry about.[1] In contrast, overlapping generations (OLG) models, such as developed by Samuelson (1958), Diamond (1965), Yaari (1965), and Blanchard (1985), allow for a simple demographic structure featuring different unlinked generations that coexist at any moment in time. In the Yaari-Blanchard framework, agents face an exogenous probability of death, equal to the birth rate, so as to yield a constant population size. Unlike the Samuelson and Diamond approach—which assumes household live for two periods, where a typical period last 35 years—the Blanchard-Yaari model is able to trace out transition dynamics at business cycle frequencies.

The paper studies both the efficiency and intergenerational distribution effects of environmental taxes. To this end, we introduce an ecological sector and endogenous labor supply in a Yaari-Blanchard model of a small open economy. The model describes four sectors: a household sector, comprising a large number of cohorts, which differ with respect to age and the level and composition of their asset portfolio, a perfectly competitive production sector, a government sector, and a foreign sector. Households derive utility from environmental quality, which is modeled as a renewable resource (for example, air or soil quality).[2] Environmental quality is negatively affected by pollution, which is generated as a side-product of capital used in production. Without government intervention, the decentralized market outcome results in too much pollution because firms fail to internalize its societal cost. Consequently, the stock of natural resources bequeathed to future generations is degraded. To correct this

[1] Altruistically linked generations view future generations as continuations of themselves, and therefore internalize any intergenerational external effects.

[2] Following John and Pecchenino (1994), John and others (1995) and Marini and Scaramozzino (1995), the environment does not feature as an input into production.

intergenerational externality, the government employs a capital income tax.

Our modeling framework is related to the work of Marini and Scaramozzino (1995), who study optimal environmental policy and intergenerational equity utilizing a continuous-time OLG model.[3] But it is most closely connected to the analysis of Bovenberg and Heijdra (1998), who study pollution taxes in the context of a closed-economy Blanchard-Yaari model, featuring an exogenously given supply of labor. Bovenberg and Heijdra's model is rather complex, particularly due to the endogeneity of the rate of interest, necessitating numerical simulations for much of the welfare analysis. Our first contribution to the literature is, therefore, to develop a much simpler model, which is capable of yielding a full set of analytical results.[4] Indeed, we consider a small open economy, which takes the rate of interest as fixed by world markets. Using the extended Blanchard-Yaari model, we study the intergenerational welfare effects of a rise in environmental taxes, assuming the government has access to lump-sum taxes and transfers to balance its budget. The private welfare effects of higher environmental taxes are shown to be unevenly distributed across generations. Environmental taxes harm old existing generations, more so the older these generations are. Current generations born close to the time of the policy change enjoy a welfare gain for low initial taxes, but future generations lose out. Environmental welfare is the same for all existing generations (due to the 'perpetual youth' assumption), but future generations gain more the later they are born after the policy change.

Next, we introduce government debt as an additional policy instrument. Public debt and the environment are related by the joint problem of intergenerational externalities. Accordingly, we design a public debt policy that can redistribute welfare across generations such that all generations enjoy the same welfare change from environmental taxes. By running a fiscal deficit initially, financed by issuing public debt, the government can transfer resources from future to current generations. Future generations, who live in a cleaner world, face higher lump-sum taxes, which are needed to retire maturing public debt. Such an egalitarian bond policy can neutralize any intergenerational externalities and generate enough political support for implementing a (constrained) first-best pollution tax.

In recent years, there has been a great deal of debate among academics and policy makers about the interaction between green policies and the tax system. Initiated by the work of Bovenberg and de Mooij (1994), attention has focused on the so-called double dividend hypothesis. It says that a green tax reform—that is, using the revenues of an increase in

[3]Mourmouras (1993), John and Pecchenino (1994) and John and others (1995), and Ono and Maeda (2001) employ the discrete-time Samuelson-Diamond model, where environmental quality is modelled as a renewable resource. Work by Howarth (1991), Howarth and Norgaard (1992), and Babu and others (1997) focuses on exhaustible resources in an intergenerational setting. This literature typically assumes that intergenerational altruism is absent. A notable exception is Jouvet, Michel and Vidal (2000).

[4]Our analysis can be seen as complementary to that of Bovenberg and Heijdra (1998). Their analysis can be interpreted as describing global coordinated environmental policy, whereas our study concerns local uncoordinated policy.

environmental taxes to cut distortionary labor taxes in a revenue-neutral fashion—may improve environmental quality and raise employment.[5] The idea is that the burden of taxation is shifted away from social desirable activities such as employment to 'public bads' such a pollution. Although various studies have employed dynamic frameworks, the literature has paid scant attention to the intergenerational dimension of the issue. By endogenizing the household's labor supply decision and recognizing pre-existing (distortionary) taxes, we are able to study the European style double dividend hypothesis and, more generally, intergenerational tax incidence issues. We show that, under plausible conditions, a double dividend materializes in the short run, but not in the long run, reflecting a fall in employment below its old steady-state level. The failure of the employment dividend is intimately linked to the slope of the short-run Laffer curve. We furthermore demonstrate that, compared to rebating revenue lump-sum, the intergenerational welfare distribution is more flattened out.

The remainder of the paper is structured as follows. Section 2 presents the model of overlapping generations for a small open economy. Section 3 analyzes the macroeconomic effects of environmental taxes assuming the government balances its budget via lump-sum transfers. Section 4 studies the intergenerational welfare effects of environmental taxes and analyzes the role of bond policy as a redistributive devise. Section 5 performs a tax reform experiment, where the revenues of environmental taxes are recycled via a cut in distortionary labor taxes. Finally, Section 6 presents some concluding remarks.

2 A Model of Perpetual Youth

2.1 Households

The economy is populated by households facing a constant probability of death, λ. Households are born at the same rate, implying a constant population size without migration. For convenience, the population size has been normalized to unity. The utility functional in period t of the representative agent of the generation born at time v is:

$$U(v,t) \equiv \int_t^{\infty} \left[\log X(v,z) + \eta \log E(z)\right] e^{(\rho+\lambda)(t-z)} dz, \tag{1}$$

where ρ denotes the pure rate of time preference, $X(v,t)$ represents 'full consumption', $E(t)$ stands for the environmental quality, and η (≥ 0) denotes the preference weight attached to the environment. Full consumption depends on goods consumption, $C(v,t)$, and labor supply, $L(v,t)$, in the following way:

$$X(v,t) \equiv C(v,t) - \left(\frac{\theta}{\theta+1}\right) L(v,t)^{\frac{\theta+1}{\theta}}, \tag{2}$$

[5] In the European literature the second dividend concerns a rise in employment, whereas in the American literature it is more generally defined as the extra benefits derived from the reduction in pre-existing distortions in the economy. In the following, we focus on the European definition. See Goulder (1995) and Bovenberg (1997) for comprehensive surveys.

where $\theta \geq 0$ represents the *intra*temporal elasticity of labor supply with respect to the wage rate. Originating from Greenwood and others (1988), this specification features the convenient property that no *inter*temporal substitution effect enters into the household's labor supply decision.[6] The agent's flow budget restriction is given by:

$$\dot{A}(v,t) = [r(t)+\lambda]\,A(v,t) + w(t)\,[1-\tau_L(t)]\,L(v,t) + Z(t) - C(v,t), \tag{3}$$

where $r(t)$ denotes the real interest rate, $\tau_L(t)$ is a labor income tax, $w(t)$ is the (age-independent) wage rate, and $Z(t)$ stands for (net) lump-sum government transfers. A dot above a variable denotes a time derivative, for example, $\dot{A}(v,t) \equiv dA(v,t)/A(v,t)$, where $A(v,t)$ is an individual's financial wealth. Since we consider a small open economy, the domestic economy is assumed to have a negligible effect on world capital markets, giving rise to the standard no-arbitrage condition, $r(t) = r$. Apart from interest income, a household's return on financial assets includes the return on actuarially fair 'reverse life-insurance' contracts.[7] Actuarial fairness, or zero profit in the insurance sector, implies that the annuity payment equals $\lambda A(v,t)$.

Maximizing household welfare (1) subject to the flow budget identity (3) and a transversality condition $\lim_{z\to\infty} A(v,z)\exp[-\int_t^z [r(s)+\lambda]\,ds] = 0$ yields expressions for the optimal time path of full consumption and labor supply:

$$\begin{aligned} X(v,t) &= (\rho+\lambda)\,[A(v,t)+H(v,t)]\,, && (4)\\ L(t) &= [(1-\tau_L(t))w(t)]^{\theta}\,, && (5)\\ \dot{X}(v,t)/X(v,t) &= r-\alpha, && (6) \end{aligned}$$

where human wealth is defined as:

$$H(v,t) \equiv \int_t^{\infty} Y_F(v,z)e^{(r+\lambda)(t-z)}dz, \tag{7}$$

where $Y_F(v,t)$ represents the net present value of disposable 'full labor income', where $r+\lambda$ is the individual's risk-of-death adjusted discount rate. Equation (4) shows that full consumption is a constant fraction of total household wealth, which consists of financial wealth and human wealth. Financial wealth is composed of government bonds, $B(v,t)$, share holdings, $V(v,t)$, and net foreign assets, $F(v,t)$, so that $A(v,t) \equiv B(v,t) + V(v,t) + F(v,t)$. Equation (5) says that optimal labor supply is determined by current net wages only and is independent of the generation index v.[8] Labor supply is exogenous if $\theta = 0$. Full consumption growth

[6]In view of the extremely tenuous empirical support existing for the intertemporal substitution effect (see Card (1994) for a recent assessment), we prefer to eliminate it from our analysis altogether.

[7]Without any bequest motive, households conclude a contract with an insurance company paying them an annuity proportional to their wealth during their life, but requiring a transfer of their entire estate upon death.

[8]Our approach thus avoids the peculiar outcome that very old (and, in this context, very rich) generations prefer to consume more leisure than their time endowment allows for, that is, they want to supply a negative amount of labor.

equals the difference between the exogenous rate of rate of interest and the pure rate of time preference. Since wages, income taxes and lump-sum transfers are generation independent, the same holds for full labor income, see (T1.11) in Table 1. This last result implies that human wealth is the same for all generations as well, that is, $H(v,t) = H(t)$.

Due to the simple demographic structure of the model, aggregate household behavior follows from integrating over currently alive individual households. Since the fraction of households born at time v still alive at time t is given by $\lambda e^{\lambda(v-t)}$, aggregate full consumption, for example, is given by $X(t) \equiv \int_{-\infty}^{t} \lambda X(v,t) e^{\lambda(v-t)} dv$. In this spirit, we further define $C(t)$ and $A(t)$ as aggregate consumption and aggregate financial wealth, respectively. Notice that the aggregates of variables that are age independent are equal to the per-generation variables. The dynamics of aggregate household behavior can now be summarized by equations (T1.1) and (T1.2) in Table 1, which we will refer to as the savings system. The first of these is the aggregate version of the household's flow budget constraint (3) and the second follows from the Euler equation for full consumption (6).[9] Note that since the reverse life-insurance is essentially a transfer from the deceased to the survivors, it plays no role at the aggregate level.

2.2 Ecology

Pollution is generated as a side-product of physical capital, $K(t)$, used in production. The flow of pollution adds to the economy's stock of pollution. Nature, however, is endowed with a regenerative capacity. Denoting the stock of pollutants by $P(t)$, we can write the emission equation as $\dot{P}(t) = \Gamma(P(t), K(t))$, where $\partial\Gamma/\partial P < 0$ and $\partial\Gamma/\partial K > 0$. The index of environmental quality featuring in the household's utility function is measured as the deviation of the stock of pollution from the 'virgin value' (P_0), $E(t) \equiv P_0 - P(t)$. Hence, environmental quality evolves over time according to:

$$\dot{E}(t) = f(E(t), K(t)), \qquad f_E < 0, f_K < 0, \tag{8}$$

where $\alpha_E \equiv -f_E > 0$ represents the regeneration speed of the natural resource and $\alpha_K \equiv (f_K/f_E)(K/E) > 0$ parameterizes the long-run ecological relationship between 'natural' capital and physical capital. Higher values of α_E imply a faster regeneration speed. In the limit, as α_E approaches infinity, environmental quality behaves more like a flow variable than a stock variable. A small value of α_K means that a large reduction in physical capital is needed in order to engineer a given rise in steady-state environmental quality.

2.3 Firms

The representative firm operates on a perfectly competitive market and produces output *net* of depreciation, $Y(t)$, according to a Cobb-Douglas production function, $Y(t) = L(t)^{\varepsilon} K(t)^{1-\varepsilon}$,

[9] We have also made use of the aggregate version of (2), (4) and (5) to derive (T1.1).

where $0 < \varepsilon < 1$, and $L(t)$ is aggregate employment. The firm faces convex installation cost, in the spirit of Uzawa (1969), which renders physical capital less mobile in the short run. The relationship between net and gross capital accumulation is specified by:

$$\dot{K}(t) = \left[\phi(I(t)/K(t)) - \delta\right] K(t), \quad \phi' > 0 > \phi'', \tag{9}$$

where $I(t)$ represents gross investment, δ denotes the rate of depreciation and $\phi(.)$ is a concave installation function. The degree of international mobility of *physical* capital can be characterized by $\sigma \equiv -(I/K)(\phi''/\phi') > 0$, where a low value of σ characterizes a high degree of capital mobility. Note that the limiting case of $\sigma = 0$ (that is, no adjustment cost) corresponds to perfect capital mobility.

The stock market value of the firm equals the net present value of its cash flow (that is, after-tax capital income minus gross investment expenses):

$$V(t) \equiv \int_t^{\infty} \left[(1 - \tau_K(z))(Y(z) - w(z)L(z)) - I(z)\right] e^{r(t-z)} dz, \tag{10}$$

where $\tau_K(t)$ is a capital income tax. The firm chooses its employment and investment plans in order to maximize $V(t)$ subject to the accumulation equation, (T1.3), and the production function, (T1.9). This gives rise to a labor demand function, (T1.6), an investment demand schedule, (T1.8), and an expression governing the evolution of the shadow value of installed capital, $q(t)$, (T1.4).[10] This last equation in conjunction with the accumulation equation describes the investment system. Consequently, the value of the firm satisfies $V(t) = q(t)K(t)$ (see Hayashi (1982)). By substituting this result in the aggregate version of the household's asset portfolio, (T1.13) is obtained.

2.4 Government and Foreign Sector

The government levies taxes on labor income and capital income, denoted by τ_L and τ_K, respectively, so that total revenue $T(t)$ is given by equation (T1.10). Government expenditure consists of lump-sum transfers to households, $Z(t)$, plus interest payments on its debt. Any fiscal deficit is financed by issuing bonds and fiscal surpluses are spent on retiring debt. The periodic budget identity of the government is given by (T1.15). A No Ponzi Game condition applies, ensuring that the government remains solvent:

$$B(t) = \int_t^{\infty} \left[T(z) - Z(z)\right] e^{r(t-z)} dz, \tag{11}$$

so that outstanding debt (left-hand side of (11)) needs to be covered by the present value of future primary surpluses (right-hand side of (11)).

[10]Note that since installation cost are homogeneous of degree zero in $I(t)$ and $K(t)$, and production is homogeneous of degree one in $K(t)$ and $L(t)$, it is straightforward to show that Tobin's marginal and average q coincide.

The goods trade balance ($TB(t)$) is defined as the difference between domestic production and absorption (that is, $C(t) + I(t)$):

$$TB(t) \equiv Y(t) - C(t) - I(t). \tag{12}$$

The current account of the balance of payments results from differentiating financial wealth:

$$\dot{F}(t) = rF(t) + TB(t). \tag{13}$$

The change in the net foreign asset position of the country equals interest income from net foreign assets plus the trade balance.

2.5 Market Equilibrium and Stability

The key equations of the model are gathered in Table 1. Equations (T1.1)-(T1.5) describe the dynamics of the model, that is, the *savings system*, (T1.1)-(T1.2), the *investment system*, (T1.3)-(T1.4), and the *ecological system* (T1.5). The static part of the model is represented by equations (T1.6)-(T1.14). Labor market equilibrium is determined by (T1.6) and (T1.7), respectively. An increase in the capital stock boosts labor demand. If labor supply is exogenous ($\theta = 0$) the employment effect of a higher capital stock vanishes. Investment demand is represented by (T1.8), the production technology is described by (T1.9), and tax revenue is given in (T1.10). Equation (T1.11) defines full income, and (T1.12)-(T1.13) relate goods consumption to full income, full consumption, and transfers. Finally, (T1.14) shows the composition of financial assets, and (T1.15) is the government's budget identity. In all our policy experiments conducted below, we ensure that government solvency is satisfied.

The dynamic part of the model can be reduced to two systems: (i) the savings system, featuring financial capital as a predetermined variable and human capital as a forward-looking variable; and (ii) the investment system, where the physical capital stock is predetermined and Tobin's q (or the shadow value of capital) is a forward-looking variable. Given that the interest rate is determined on the world market, the latter is also the only "price variable" available to clear the capital market. The model is locally saddle-point stable within certain bounds as summarized by Proposition 1.

Proposition 1 *The model is locally saddle-point stable if $\rho < r < \rho + \lambda$. The system can be decomposed in two systems, featuring the following properties:*
(i) the investment system has two distinct characteristic roots, $-h_I < 0$, and $r_I = r + h_I > r$, where the stable root satisfies $\partial h_I/\partial\sigma < 0$ and $h_I \to \infty$ if $\sigma \to 0$; and
(ii) the savings system has two distinct characteristic roots, $-h_S = r - (\rho + \lambda) < 0$ and $r_S = r + \lambda > 0$, where the stable root satisfies $\partial h_I/\partial\lambda > 0$ and $h_S \to \infty$ if $\lambda \to \infty$.

PROOF*: See Heijdra, Kooiman and Ligthart (2004).*

Table 1. Summary of the Model

$$\dot{A}(t) = [r - (\rho + \lambda)]\, A(t) - (\rho + \lambda) H(t) + Y_F(t), \tag{T1.1}$$
$$\dot{H}(t) = (r + \lambda) H(t) - Y_F(t), \tag{T1.2}$$
$$\dot{K}(t) = [\phi(I(t)/K(t)) - \delta]\, K(t), \tag{T1.3}$$
$$\dot{q}(t) = [r + \delta - \phi(I(t)/K(t))]\, q(t) + I(t)/K(t) - [1 - \tau_K(t)]\,(1 - \varepsilon)(L(t)/K(t))^{\varepsilon}, \tag{T1.4}$$
$$\dot{E}(t) = f(E(t), K(t)), \tag{T1.5}$$
$$w(t) = \varepsilon(L(t)/K(t))^{\varepsilon - 1}, \tag{T1.6}$$
$$L(t) = [w(t)(1 - \tau_L(t))]^{\theta}, \tag{T1.7}$$
$$1 = q(t)\phi'(I(t)/K(t)), \tag{T1.8}$$
$$Y(t) = L(t)^{\varepsilon} K(t)^{1-\varepsilon}, \tag{T1.9}$$
$$T(t) = \tau_K(t)\,[Y(t) - w(t)L(t)] + \tau_L(t) w(t) L(t), \tag{T1.10}$$
$$Y_F(t) = (1 + \theta)^{-1}\,[w(t)(1 - \tau_L(t))]^{1+\theta} + Z(t), \tag{T1.11}$$
$$C(t) = X(t) + \theta\,[Y_F(t) - Z(t)]\,, \tag{T1.12}$$
$$X(t) = (\rho + \lambda)\,[A(t) + H(t)]\,, \tag{T1.13}$$
$$A(t) = q(t)K(t) + B(t) + F(t), \tag{T1.14}$$
$$\dot{B}(t) = rB(t) + Z(t) - T(t), \tag{T1.15}$$

The assumption made regarding the world interest rate serves to ensure that agents save financial wealth (the lower bound, $r > \rho$) and that a steady state exists (the upper bound, $\rho + \lambda > r$). The latter guarantees the validity of the log-linearization of the model around a given steady state (see the Appendix Table). Note that the presence of installation cost in the model results in sluggish adjustment in the capital stock and the emergence of transitional dynamics.[11] The adjustment speed of the investment system and savings system is denoted by h_I and h_S, respectively.

3 Dynamic Allocation Effects of Environmental Taxes

In this section, we study the macroeconomic dynamics of an unanticipated and permanent increase in an environmentally-motivated capital income tax. We refer to this as an environmental tax, because it acts directly on the source of pollution (that is, capital use), in line

[11]Without installation costs, deviations from the world interest rate would give rise to an infinite rate of investment during an infinitesimally small time interval.

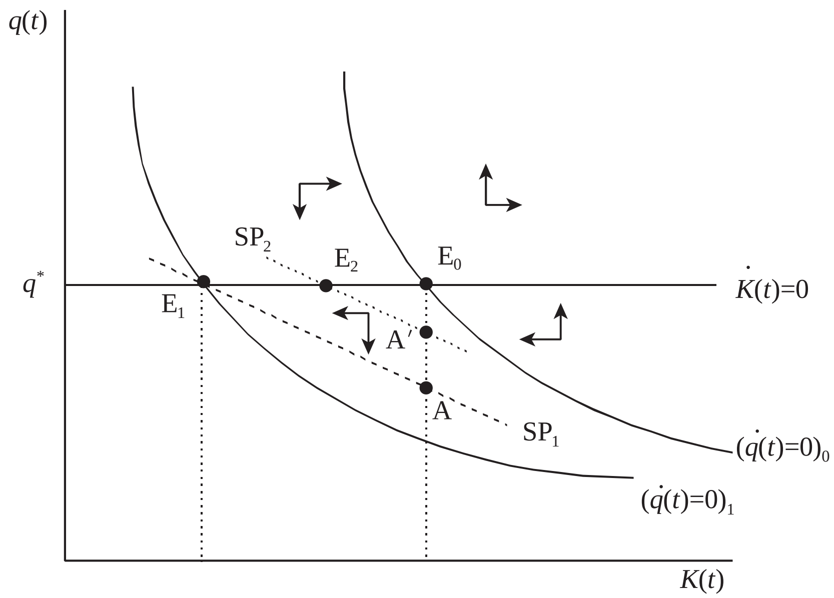

Figure 1: Dynamics of the Investment System

with Dixit's (1985) 'targeting principle.' The labor income tax is held constant at this stage and bond policy is temporarily abstracted from. Collected tax revenue is returned to households via lump-sum transfers so as to keep the government budget balanced at each instant of time.[12] The formal proofs underlying the qualitative analysis—obtained by log-linearizing the model around an initial steady state and subsequently perturbing the system—can be found in Table 2 and Heijdra, Kooiman and Ligthart (2004). Appendix A.1 sets out the notational conventions employed throughout.

3.1 Graphical Apparatus

The investment system can be summarized by a simple diagrammatic apparatus (Figure 1). The $\dot{K}(t) = 0$ locus represents (q, K)-combinations for which the capital stock is in equilibrium, that is, for which net investment is zero. It is horizontal at q^*, which denotes the unique value of Tobin's q for which $\phi(.) = \delta$ (see (T1.3) and (T1.8)). Values of $q(t)$ that are larger (smaller) than q^*, yield positive (negative) net investment as is indicated by the horizontal arrows in Figure 1.

The $\dot{q}(t) = 0$ locus shows (q, K)-combinations for which Tobin's q is constant over time. It is downward sloping because a higher capital stock leads to a fall in the marginal product of capital and thus yields lower dividends to share owners. For points to the right (left) of the $\dot{q}(t) = 0$ line the marginal product of capital is too low (high), giving rise to capital gains

[12]Keeping the labor tax constant has the important implication that revenue recycling via net lump-sum transfers only affects the savings system.

(losses) on investment. Hence, $\dot{q}(t) > 0$ (< 0) to the left (right) of the line, as has been shown with vertical arrows in Figure 1. Not surprisingly, in view of the discussion surrounding Proposition 1, the arrow configuration confirms that the equilibrium at E_0 is saddle point stable.

The degree of physical capital mobility, as parameterized by σ, is an important determinant of the transition path of the economy after a shock. Indeed, the lower is σ, the more mobile is capital, the more approximate is the saddle path to the $\dot{K}(t) = 0$ line, and the higher is the adjustment speed h_I. In the limiting case of $\sigma \rightarrow 0$ (that is, perfect mobility), the saddle path (SP) coincides with $\dot{K}(t) = 0$ line. In that case, transition to the new steady state is immediate ($h_I \rightarrow \infty$), reflecting that capital is now a jump variable. Accordingly, Tobin's q is equal to unity, precluding any capital gains or losses.

3.2 Raising Environmental Taxes

The increase in the environmental tax reduces the after-tax marginal product of capital and thus also lowers Tobin's q, which is the discounted value of present and future marginal products of capital. To restore the equilibrium value of Tobin's q, the before-tax marginal product of capital must rise, that is, the capital stock must fall. As a result, the $\dot{q}(t) = 0$ line shifts to the left. Given that there is no long-run effect on Tobin's q, the steady state shifts from E_0 to E_1, yielding a lower long-run capital stock. The fall in the steady-state capital stock is particularly large if labor supply is very elastic (large θ) and the labor share in production is low (small ε). A decrease in the capital stock also reduces the long-run level of employment, reflecting the cooperative nature of the two factors of production. Consequently, wages in the long run fall too, owing to the lower labor productivity. Table 2 summarizes the impact effects (at $t = 0$ when the policy is implemented) and steady state effects (at $t \rightarrow \infty$) on the main economic variables.

Tobin's q jumps down immediately when the policy change is implemented, thereby hurting capital owners, who experience a drop in financial wealth. Intuitively, the value of the installed capital drops, reflecting the prospect of the after-tax rate of return to capital being below the fixed world interest rate during transition. In terms of Figure 1, the economy moves from E_0 to point A on the saddle path SP_1. Capital is not affected at impact, explaining why employment, wages, output, and environmental quality do not respond either (see (A.1)-(A.3) in the Appendix). Full income rises because no reallocations of resources have taken place yet and the only effect is a positive tax rate effect, which increases tax revenue and the concomitant government transfers. Human wealth—defined as the net present value of the whole future path of full income—displays a jump at impact if initial taxes are low or the *generational turnover effect* (cf. Heijdra and Ligthart (2000)) is strong or both (see (T2.8a)).[13]

[13]Human capital is likely to rise at impact if generational turnover is much larger than the adjustment speed

During transition, gross investment collapses while the existing capital stock keeps depreciating, causing the capital stock to fall gradually. In terms of Figure 1, the economy moves along the saddle path from A to E_1 according to $\tilde{K}(t) = e^{-h_I t}\tilde{q}(0) < 0$. Capital losses fall with the height of the adjustment speed of the investment system, owing to a faster restoration of the after-tax marginal product of capital. The transition paths of employment, wages, and output mimic that of the capital stock; all variables show a gradual decrease to their new steady state values. Decumulation of capital decreases the marginal product of labor, shifting the labor demand curve inward, thereby reducing employment and lowering wages. Indeed, the full long-run burden of environmental taxes is shifted to labor in a small open economy.[14] Consequently, full income exclusive of transfers falls. Both the reduction in the capital stock and the induced fall in employment reduce output during transition, which erodes the tax base so that tax revenue and government transfers also fall. Therefore, full income inclusive of transfers falls even more than full income exclusive of transfers.[15] Not surprisingly, the transition paths for human and financial wealth display a pattern similar to that of full income.[16]

We saw that at impact there is no effect on the environment. During transition, the gradual fall in the capital stock causes a monotonic increase in environmental quality. In the long run, environmental quality improves, reflecting: (i) lower pollution associated with capital decumulation; and (ii) resource regeneration connected with the fall in the capital stock. If capital causes a lot of pollution—and thus has a strong impact on ecological regeneration (that is, α_K large)—a reduction in capital translates into a large increase in environmental quality. On the other hand, if $\alpha_K = 0$ capital does not generate any pollution and a reduction of it does not have any impact on environmental quality (see (T2.9b)).

4 Intergenerational Welfare Effects of Environmental Taxes

This section studies in what way a rise in the environmental tax affects intergenerational welfare. Again, labor taxes are held constant and bond policy is abstracted from in Sections 4.1-4.2. Throughout, we make use of the additive nature of the households' utility functional (1) by considering separately the changes in non-environmental (or private component of) welfare (shorthand 'P') and environmental welfare (shorthand 'E'). Then, we can write:

of the investment system. Intuitively, at a low value of h_I (relative to λ) capital decumulation is sluggish so that full income falls only slowly, whereas at a high value of λ the long run decline in full labor income is heavily discounted as it shifts past the horizons of the currently living generations. See Heijdra, Kooiman and Ligthart (2004).

[14] See also Bovenberg (1993) and Heijdra and Ligthart (2000).

[15] Equation (T2.6b) shows that full income does not change if: (i) the initial capital income tax is zero ($\tau_K = 0$); and (ii) either labor supply is exogenous ($\theta = 0$) or the labor income tax is zero (τ_L).

[16] Note that the more sluggish the investment system or savings system is, the larger is the ultimate reduction in financial wealth.

$dU(v,t) \equiv dU_P(v,t) + \eta dU_E(t)$, where η is the weight given to the environmental component. To build some intuition, we first study the standard case where households are infinitely lived (that is, $\lambda = 0$) and then move on to the more complex case of finitely-lived households (that is, $\lambda > 0$).[17]

4.1 Infinitely-Lived Households

If households face a zero probability of death the rate of interest should equal the pure rate of time preference to guarantee the existence of an initial steady state. Then, perturbing the system by increasing the environmental tax yields the change in private welfare:

$$dU_P(0) = \left(\frac{h_I}{\rho + h_I}\right) \frac{\tilde{Y}_F(\infty)}{\rho \omega_X} \leq 0, \tag{14}$$

where ω_X denotes the share of full consumption in national income and $\tilde{Y}_F(\infty) \equiv dY_F(\infty)/Y$ denotes the income-scaled change in full income in the new steady state. Equation (14) shows that for a given decline in full income, households are worse off the faster the economy approaches its new steady state (a large h_I) and the smaller the pure rate of time preference (ρ). In the latter case, agents discount future losses more heavily so that any change in full income has less of an effect on life-time utility. If initial labor and environmental tax rates are zero—so that taxes do no distort agents' decisions—full consumption is unaffected and so is private welfare. In that case, the increase in environmental taxes has no first-order effects, because the cost of the fall in the capital stock are exactly offset by the revenue raised by environmental taxes. If initial tax rates are positive, however, cost and benefits are equalized in after-tax terms, implying inequality in pre-tax terms. Agents reduce the stock of capital more than is socially desirable, because they fail to internalize the effect of the erosion of the tax base on government transfers that supplement their labor earnings.

The environmental component of utility rises, owing to an improvement in environmental quality connected with the fall in the capital stock:

$$dU_E(0) = -\left(\frac{h_I}{\rho + h_I}\right)\left(\frac{\alpha_E}{\rho + \alpha_E}\right)\left(\frac{\alpha_K}{\rho}\right) \tilde{K}(\infty) > 0. \tag{15}$$

A reduction in either h_I or α_E results in slower growth of the natural resource and causes a lower net present value of future environmental benefits. Environmental welfare, however, increases with α_K, since it amplifies the positive environmental effect of a given change in the capital stock.

Whether or not the gain in environmental utility compensates for the loss of private welfare depends on the parameter setting and the initial (distortionary) tax rates. Except for the case of zero initial taxes, where the environmental tax does not generate any deadweight losses, there is no unambiguous answer. With initial taxes positive, we can solve $dU(\tau_K, \tau_L)/d\tau_K = 0$

[17] The derivation of the welfare changes can be found in Heijdra, Kooiman and Ligthart (2004).

to arrive at the optimal second-best environmental tax as a function of the exogenously given labor income tax:

$$\tau_K^*(\tau_L) = \eta \left(\frac{\alpha_E}{\rho + \alpha_E} \right) \left(\frac{\alpha_K \omega_X}{1 - \varepsilon} \right) - \left(\frac{\varepsilon \theta}{1 + (1 - \varepsilon)\theta} \right) \tau_L, \tag{16}$$

where an asterisk indicates an 'optimal' value. The first term of (16) being the Pigovian tax rate and the second term is the tax interaction term (that is, the distortionary effect of labor taxation). The latter drops out if the initial labor income tax rate is zero or labor supply is perfectly inelastic. Note that (16) is a quasi-reduced form since the right-hand side features ω_X, which is a function of initial tax rates. Still we can find comparative static results for any model parameter not affecting ω_X, notably the parameters affecting the ecological side of the model. Strong concern for the environment (large η), a high regeneration speed relative to the rate of time preference (large α_E and small ρ), and a high pollution-generating effect of capital (α_K large) all lead to a higher optimal pollution tax.

4.2 Finitely-Lived Households

We now turn to finitely-lived households—and thus study the full, unrestricted model—and evaluate the welfare consequences of an increase in environmental taxes. Since an agent's stock of financial wealth differs with his age, environmental taxes affect generations' capital income differently. Newborns do not own any financial wealth yet, and thus a drop in Tobin's q would not affect them. Old existing generations, however, who have accumulated capital all their life are seriously hurt by the policy change.

As above, we first discuss the effects on private welfare. Because this component of utility is age dependent, we distinguish between generations that are alive at the time of the policy change (denoted by $t = 0$) and those generations that are born after the shock (denoted by $v = t > 0$, where the generation index coincides with the historical time index). The path of welfare change of existing generations at $t = 0$ as a function of their generation index v is shown to be

$$dU_P(v, 0) = (1 - \alpha_H(v)) dU_P(-\infty, 0) + \alpha_H(v) dU_P(0, 0), \; v \leq 0, \tag{17}$$

a weighted average of the effect on an extremely old generation (that is, $dU_P(-\infty, 0)$) and the effect on a newly-born generation (that is, $dU_P(0, 0)$), where the weighing factor, $\alpha_H(v) \equiv e^{(r-\rho)v}$, is the share of human wealth in total wealth of an agent belonging to generation v. A young agent (with $v \to 0$), consumes out of human wealth only (so that $\alpha_H \to 1$).

Very old generations and newly born generations experience opposite changes in private welfare:

$$dU_P(-\infty, 0) = \frac{\tilde{A}(0)}{(\rho + \lambda)\omega_A} < 0, \quad dU_P(0, 0) = \frac{\tilde{H}(0)}{(\rho + \lambda)\omega_H} \gtrless 0, \tag{18}$$

where ω_A is the share of (financial) asset income in national income and $\omega_H \equiv rH/Y$ denotes the share of human capital income in national income. Very old agents suffer from the environmental tax increase, because they primarily consume out of capital income. Whatever the initial tax rates are, capital owners experience a welfare loss proportional to the capital loss at impact, $\tilde{A}(0) > 0$. Young generations' welfare change, however, is proportional to the change in human wealth at impact, which is positive if initial taxes are low or the generational-turnover effect is strong or both (as was shown in Section 3). Not surprisingly, financial wealth does not play a role here, reflecting that each generation is born without any.

The private welfare consequences for future generations ($t \geq 0$) evaluated at birth can similarly be written as a weighted average of the effect on a newly-born generation and on someone born in the new steady state:

$$dU_P(t,t) = e^{-h_I t} dU_P(0,0) + \left(1 - e^{-h_I t}\right) dU_P(\infty,\infty) = \frac{\tilde{H}(t)}{(\rho+\lambda)\omega_H}, \tag{19}$$

where the weights are determined by the adjustment speed of the investment system. Alternatively, the change in non-environmental welfare can also be expressed as being proportional to the transition path for human wealth (the second part of (19)). As it turns out the expression for the welfare loss of newborns generalizes to future generations. Financial capital plays no role, and the reasoning is identical to that for generation zero's welfare change. Note that generations born in the new steady state unambiguously suffer in the presence of pre-existing taxes. Intuitively, full income by then has declined and so has human wealth, the determinant of their welfare.

We now consider the change in environmental welfare in order to explore whether or not the losses we found for about every generation (except for generations born at time of the shock) are in some way compensated for by a 'green dividend.' The environmental welfare change for the generation born at time t can be written as a weighted average of the welfare change of those alive at the time of the shock and those born in the new steady state:

$$dU_E(t) = \left[1 - a(h_I, \alpha_E, \rho+\lambda, t)\right] dU_E(0) + a(h_I, \alpha_E, \rho+\lambda, t) dU_E(\infty), \tag{20}$$

where $a(h_I, \alpha_E, \rho+\lambda, t)$ is an adjustment term which is increasing in t and lies between zero and one.[18] The adjustment term represents the part of the evolution of the stock of natural resources that has been completed at the time of birth of the generation at hand. For generations alive at $t = 0$ this share is zero so that welfare changes according to:

$$dU_E(0) = -\left(\frac{h_I}{\rho+\lambda+h_I}\right)\left(\frac{\alpha_E}{\rho+\lambda+\alpha_E}\right)\left(\frac{\alpha_K}{\rho+\lambda}\right)\tilde{K}(\infty) > 0. \tag{21}$$

Compared with infinite horizons, the modification introduced by overlapping generations is that the rate of time preference, ρ, is replaced by the risk-of-death adjusted discount rate,

[18]Further details can be found in Heijdra, Kooiman and Ligthart (2004).

$\rho+\lambda$. Intuitively, in discounting the future agents take into account the probability that they are not alive to enjoy a cleaner environment.

The change in environmental welfare of a generation born in the new steady state is:

$$dU_E(\infty) = -\left(\frac{\alpha_K}{\rho+\lambda}\right)\tilde{K}(\infty) > 0. \tag{22}$$

By the time the new steady-state generations are born, environmental quality is at a higher level compared to the level for those alive at $t = 0$.[19] The environmental welfare gain rises with $-\alpha_K/(\rho+\lambda)$ for a given decline in the capital stock. It follows immediately from (20)-(22) that environmental welfare rises monotonically over time so that generations born in the new steady state enjoy the largest environmental welfare gain. The effects are summarized in Proposition 2.

Proposition 2 *A rise in environmental taxes of which the revenue is returned to households in a lump-sum fashion yields the following welfare changes:*
(i) $dU_P(-\infty,0) < 0$*;*
(ii) $dU_P(0,0) > 0$ *for low initial tax rates;*
(iii) $dU_P(\infty,\infty) \leq 0$;
(iv) $dU_P(-\infty,0) < dU_P(0,0)$ *for low initial tax rates;*
(v) $dU_P(\infty,\infty) < dU_P(0,0)$ *for all initial tax rates; and*
(vi) $0 < dU_E(0) < dU_E(\infty)$ *for all initial tax rates.*

PROOF: *See Heijdra, Kooiman and Ligthart (2004).*[20]

Environmental welfare rises so that indeed the losses in private welfare are (at least partially) offset by a green dividend. The extent to which total welfare of each generation rises depends on the parameter configuration, primarily on η of course. In sum, the redistributive effects of the environmental tax increase can roughly be characterized as follows. First, if initial taxes are not too high, the oldest generations are the ones that suffer most. Second, generations born right after the tax increase are best off owing to the temporary rise in labor earnings. Generations born near the new steady state enjoy a welfare gain only for high values of η, that is, a strong concern for the environment. In that case, the increase in environmental quality more than offsets the drop in their human wealth.

[19] Note that the two factors—that is, $\frac{h_I}{\rho+\lambda+h_I} < 1$ and $\frac{\alpha_E}{\rho+\lambda+\alpha_E} < 1$, representing the effects of discounting on the transition path—disappear from (22), leaving the steady-state effects only.

[20] It should be noted that inequality (iv) requires weaker conditions than inequality (ii) since the latter is a sufficient condition for the former. However, it is not a necessary condition because if (ii) does not hold (iv) may very well be true.

4.3 Redistributive Bond Policy

The previous sections have shown that environmental taxes generate efficiency gains, owing to the internalization of intergenerational pollution externalities. Such a policy, however, redistributes welfare away from existing generations toward newly-born generations, generating non-unanimous political support among generations. Indeed, environmental taxes may even be obstructed if political decisions are based on majority voting. Accordingly, there is a need for an accompanying policy instrument to distribute the efficiency gains more evenly across generations. Bond policy could serve this purpose as it generates intergenerational fiscal externalities. By initially running a fiscal deficit, financed by issuing long-term government bonds, the government can make transfer payments to existing generations. Consequently, future generations, who live in a cleaner world, are confronted with higher lump-sum taxes necessary to finance debt redemption.

We now introduce government debt in our analysis and show that an egalitarian bond policy exists that renders the welfare change generation independent:

$$dU(v,0) = dU(t,t) = \frac{\Pi}{\rho + \lambda} > 0, \quad (-v, t \geqslant 0), \tag{23}$$

where Π represents the common efficiency gain resulting from the pollution tax increase *cum* public debt policy. We postulate a path of government debt:[21]

$$\tilde{B}(t) = b_0 + b_1 e^{-\phi_1 t} + b_2 e^{-\phi_2 t}, \tag{24}$$

so as to yield an outcome as described in (23). The sum of coefficients, $b_0 + b_1 + b_2$, parameterizes the change in the initial debt stock (that is, $\tilde{B}(0)$), $b_0 = \tilde{B}(\infty)$ reflects the long-run change in government debt, and ϕ_i (with $i = \{1, 2\}$) determines the adjustment speed.

We determine the appropriate bond policy as follows. Combining (24) and the government budget identity ((AT.15) in the Appendix Table) yields the path of primary government deficits $\tilde{D}(t) \equiv \tilde{Z}(t) - \tilde{T}(t)$ as a function of the parameters ϕ_i and b_j, where $j = \{0, 1, 2\}$. Since the investment system is independent of the path of government debt, full income under bond policy can be written as:

$$\left[\tilde{Y}_F(t)\right]_B \equiv \left[\tilde{Y}_F(t)\right]_{LS} + \tilde{D}(t), \tag{25}$$

where the subscript 'B' refers to bond policy, 'LS' refers to the lump-sum rebating scheme analyzed in Section 3 and $[\tilde{Y}_F(t)]_{LS}$ is reported in the Appendix (see (A.7)). Intuitively, the use of bond policy allows us to disentangle the paths of transfers and tax revenue, subject of course to the government's solvency condition.

The next step is to solve the savings system for financial and human wealth, both of which are affected by the parameters of the bond path. By confronting the solutions with

[21] We use the method of undetermined coefficients to solve for the egalitarian policy. Note that government solvency is ensured provided $\phi_i \geq 0$ and $| b_j | \ll \infty$.

(23), expressions for the parameters of the bond path (see below) and the efficiency gain from environmental taxes can be obtained. The latter reads as:

$$\Pi = \left(\frac{h_I}{r+h_I}\right)\left[(1/\omega_X)\left[\tilde{Y}_F(\infty)\right]_{LS} + \eta\left(\frac{\alpha_E}{r+\alpha_E}\right)\left[\tilde{E}(\infty)\right]_{LS}\right], \tag{26}$$

so that the efficiency gain rises with $[\tilde{Y}_F(\infty)]_{LS}$ and $[\tilde{E}(\infty)]_{LS}$, both of which are unaffected by the bond path. Note that Π may turn out to be negative—indicating an efficiency loss—if the long-run decline in full income dominates the rise in environmental quality. In that case, initial taxes are already too high.[22] The effect of $[\tilde{Y}_F(\infty)]_{LS}$ on Π depends on the adjustment speed of the investment system and the share of full consumption in income. The effect of the rise in $[\tilde{E}(\infty)]_{LS}$ on the efficiency gain is positively affected by the regeneration speed of the renewable resource and the preference weight attached to environmental quality.

Not surprisingly, the efficiency gain derived above coincides with the gain derived for the infinite horizon case (compare (14)-(15) and (26)). Without initial labor income taxes, a policy of increasing environmental taxes accompanied by an egalitarian bond policy is capable of taking the economy to a first-best outcome. Intuitively, environmental taxes can be set so as to fully internalize the pollution externality, whereas public transfers address the intertemporal distortion arising from overlapping generations.

We now characterize the parameters of the bond path. Simultaneously with the tax increase, the government adjusts its debt stock upward, yielding additional funds to the government:

$$\begin{aligned}\tilde{B}(0) \quad &= \quad \left(\frac{r}{r+h_I}\right)\left[\tilde{Y}_F(0)\right]_{LS} \\ &\quad + \omega_A\left[\Pi - \eta\left(\frac{h_I}{\rho+\lambda+h_I}\right)\left(\frac{\alpha_E}{\rho+\lambda+\alpha_E}\right)\left[\tilde{E}(\infty)\right]_{LS}\right].\end{aligned} \tag{27}$$

The leading term on the right-hand side of (27) represents the rise in full consumption, and the second term reflects the difference between the efficiency gain and the change in environmental welfare weighted by the financial wealth share. If $\tilde{B}(0) > 0$, the government provides a one-off subsidy to capital owners in order to compensate them for the capital loss on their assets. Accordingly, all present generations experience the same welfare gain.[23] The long-run debt position amounts to:

$$\left(\frac{r}{r+\lambda}\right)\tilde{B}(\infty) = \left(\frac{r}{r+\lambda}\right)\left[\tilde{Y}_F(\infty)\right]_{LS} - \omega_H\left[\Pi - \eta\left[\tilde{E}(\infty)\right]_{LS}\right]. \tag{28}$$

The left hand side of (28) represents the net present value of the steady state change in the government's interest liabilities. Note the congruence between (27) and (28). The first term

[22] Without pre-existing taxes, the gain is unambiguously positive because full income returns to its initial steady-state value in the long run. See (T2.5b) in Table 2.

[23] Indeed, inspecting (17)-(18) reveals that egalitarian policy eliminates the generation-specific term in (17), so that $[\tilde{A}(0)/\omega_A]_B$ must equal $[\tilde{H}(0)/\omega_H]_B$.

on the right-hand side of (28) is the fall in long-run human wealth in the absence of bond policy, and the second term reflects the difference between the efficiency gain and the change in environmental welfare expressed in terms of human wealth. Since the environmental gain materializes only to its full extent in the new steady state, the bond policy intertemporally reallocates the private cost of this improvement to future generations by retiring debt in the future, which was built up at the time of the shock. In that way, those who are living in a cleaner world face higher lump-sum taxes. The three equations above specify the bond path only partially, a full derivation can be found in Heijdra, Kooiman and Ligthart (2004).[24]

5 Environmental Taxes Under Labor Tax Recycling

In this section, we consider a green tax reform that consists of an unanticipated and permanent increase in the environmental tax combined with a reduction in the distortionary labor income tax. We will call this scenario labor tax recycling. To allow for a comparison with lump-sum tax recycling, we require the tax reform to be budgetary neutral on impact, but also during transition. Lump-sum transfers and bond policy are ruled out, implying that the labor tax is the only tax variable that can be endogenously varied over time to keep the government's budget balanced. We start with an analysis of a time varying labor income tax rate and derive conditions on initial tax rates, yielding an equilibrium on the 'correct side' of the Laffer curve. Subsequently, we analyze the macroeconomic and distributional consequences of the policy and compare it with lump-sum rebating.

5.1 Time-Varying Labor Income Taxation

If the government aims to keep its budget balanced at each instant of time, one of the tax rates needs to vary over time. Intuitively, the tax reform affects the base of both taxes during transition, implying that tax revenue would vary if both tax rates were kept constant after the implementation of the reform. Without bond policy or lump-sum taxes this contradicts the presupposed budget neutrality of the tax reform. Accordingly, if environmental taxes are chosen to be kept constant, we have to allow for a time-varying labor income tax rate. This apparent analytical complication does not, however, prevent us from solving the model in almost the same manner as before. The only modification required is that we first solve for the time-varying labor income tax rate as a function of the environmental tax rate and the capital stock. Subsequently, the result can be plugged into the investment system which can then be solved as before.

The path of the labor income tax rate that satisfies the conditions posed is found by setting $\tilde{B}(t) = \tilde{Z}(t) = \tilde{T}(t) = 0$ (so that (AT.15) holds trivially) and solving (AT.10) for

[24] For completeness we state that the adjustment speed parameters are $\phi_1 = h_I$ and $\phi_2 = \alpha_E$, which is rather obvious from the observation that the path of private welfare must smooth the path of environmental welfare so as to yield an equal welfare change across all generations.

$\tilde{\tau}_L(t)$:

$$\varepsilon(1-\tau_L)\tilde{\tau}_L(t) = -\left[(1-\varepsilon)(1-\tau_K)\tilde{\tau}_K + \omega_Z \tilde{Y}(t)\right], \tag{29}$$

where $\omega_Z \equiv \varepsilon\tau_L + (1-\varepsilon)\tau_K \geq 0$ is the share of tax revenue in national income. The two terms on the right-hand side of (29) represent the tax rate effect and the tax base effect, respectively. In view of the Cobb-Douglas production structure, the tax bases for labor and capital are both proportional to aggregate output. Of course, if initial taxes are zero, that is, $\tau_L = \tau_K = 0$ so that $\omega_Z = 0$, there is no tax base effect to worry about and thus only the tax rate effect remains. If labor supply is exogenous ($\theta = 0$), the labor tax does not affect employment, implying that the short-run tax base effect is zero (as capital is predetermined, $\tilde{K}(0) = 0$) and the long-run tax base effect is proportional to the long-run effect on the capital stock (see (A.3)).

Generally, $\theta > 0$ and $\omega_Z > 0$, so that the tax base effect in the impact period is induced by labor market dynamics only, and by labor and capital reallocation during transition and in the long run. The implied path for the labor tax can be computed by substituting the quasi-reduced form for output (A.3) into (29):

$$\tilde{\tau}_L(t) = -\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\frac{(1+\theta)\omega_Z \tilde{K}(t) + [1+\theta(1-\varepsilon)](1-\tau_K)\tilde{\tau}_K}{1-\tau_L(1+\theta)+\theta(1-\varepsilon)(1-\tau_K)}\right]. \tag{30}$$

If initial taxes are zero, the term in square brackets on the right-hand side of (30) is unambiguously positive, so that a rise in the environmental tax yields a fall in the labor tax rate. As a result, the economy operates on the upward sloping segment of the Laffer curve. Obviously, this result does not extend to all initial tax rates. High initial tax rates may cause a severe erosion of the labor and capital tax bases such that the economy ends up on the wrong side of the Laffer curve. To ensure an equilibrium on the upward-sloping segment of the Laffer curve, we make a number of assumptions on initial tax rates:

Assumption 1 *The initial tax rates τ_K and τ_L satisfy:*[25]

$$[\tilde{\tau}_L(0)/\tilde{\tau}_K < 0] \quad : \quad 1-\tau_L(1+\theta)+\theta(1-\varepsilon)(1-\tau_K) > 0, \tag{SRL}$$

$$[\tilde{\tau}_L(\infty)/\tilde{\tau}_K < 0] \quad : \quad \varepsilon\left[1-\tau_L(1+\theta)\right] - \left[1+\theta(1-\varepsilon)\right]\tau_K > 0, \tag{LRL}$$

implying that the initial tax rates are such that the economy operates on the upward-sloping segment of the Laffer curve in both the short and long run.

[25] Heijdra, Kooiman and Ligthart (2004) show that there is always a non-trivial region in the south-west of the feasible tax square where these conditions hold. Specifically, if $\theta > 0$, the region satisfying the short-run Laffer curve condition (SRL) is the triangle defined by $(\tau_L, \tau_K) = (0,0)$, $(\tau_L, \tau_K) = (0, \varepsilon/(\theta(1-\varepsilon)))$, and $(\tau_L, \tau_K) = (1/(1+\theta), 0)$. The region satisfying the long-run Laffer-curve condition (LRL) is the (smaller) triangle defined by $(\tau_L, \tau_K) = (0,0)$, $(\tau_L, \tau_K) = (0, \varepsilon/(1+\theta(1-\varepsilon)))$, and $(\tau_L, \tau_K) = (1/(1+\theta), 0)$. If $\theta = 0$ SRL, holds for all tax rates.

Before studying the macroeconomic and distributional consequences of the tax reform, we must reconsider the stability of the investment system because the reform affects its dynamics. More specifically, the investment system is no longer stable for all levels of initial taxes. Intuitively, stability requires the after-tax marginal product of capital to rise if the capital stock falls. By plugging (30) into the quasi-reduced form expression for output in (A.3), we obtain an expression for the after-tax marginal product of capital:

$$\tilde{Y}(t) - \tilde{K}(t) - \tilde{\tau}_K = -\psi(\tau_K, \tau_L)\tilde{K}(t) - \chi(\tau_K, \tau_L)\tilde{\tau}_K, \tag{31}$$

where the parameters $\psi(\tau_K, \tau_L)$ and $\chi(\tau_K, \tau_L)$ are defined as follows:

$$\begin{aligned} \psi(\tau_K, \tau_L) &\equiv \frac{\varepsilon\left[1 - \tau_L(1+\theta)\right] - \theta(1-\varepsilon)\tau_K}{1 - \tau_L(1+\theta) + \theta(1-\varepsilon)(1-\tau_K)}, \\ \chi(\tau_K, \tau_L) &\equiv \frac{1 - \tau_L(1+\theta)}{1 - \tau_L(1+\theta) + \theta(1-\varepsilon)(1-\tau_K)}. \end{aligned} \tag{32}$$

The short-run Laffer-curve condition (SRL) ensures that the denominator of $\psi(\tau_K, \tau_L)$ and $\chi(\tau_K, \tau_L)$ is positive. Stability is achieved provided the numerator of $\psi(\tau_K, \tau_L)$ is positive too, yielding the stability condition (SC):

Assumption 2 *The initial tax rates τ_K and τ_L satisfy:*

$$[\psi(\tau_K, \tau_L) > 0] : \varepsilon\left[1 - \tau_L(1+\theta)\right] - \theta(1-\varepsilon)\tau_K > 0, \tag{SC}$$

that is, the initial tax rates are such as to preserve macroeconomic diminishing returns to capital.

Clearly, this assumption holds if initial tax rates are zero or if labor supply is exogenous. It furthermore precludes (local) fiscal increasing returns to capital, which were to exist if the labor tax cut creates such a large increase in labor supply (and thus employment) that after-tax capital productivity is increasing in the capital stock. This outcome is clearly unstable. We have to modify Proposition 1 to yield:

Proposition 3 *Suppose that $\rho < r < \rho + \lambda$, $\theta > 0$, and that Assumptions 1 and 2 hold, then the full model is locally saddle-point stable. The subsystems can be characterized as follows: (i) the investment system has distinct characteristic roots, $-h_I < -h_I^* < 0$ and $r < r_I^* = r + h_I^* < r_I = r + h_I$, where the stable root satisfies $\partial h_I^*/\partial\sigma < 0$ and $h_I^* \to \infty$ as $\sigma \to 0$; and (ii) the savings system has distinct characteristic roots, $-h_S = r - (\rho + \lambda) < 0$, and $r_S = r + \lambda > 0$.*

Proof*: See Appendix A.3.*

The dynamic properties of the economy are similar under lump-sum rebating and green tax reform, but the adjustment speed is the lowest under the latter scenario, that is, $h_I > h_I^*$. This has important repercussions for both the macroeconomic and distributional effects of the environmental tax.

5.2 Dynamic Allocation Effects

Figure 1 can be used again to describe the effects of a rise in environmental taxes of which the revenues are used to cut labor taxes. The shift to the left in the $\dot{q}(t)$ line is shown to be smaller than under lump-sum tax recycling. As a result, the long-run equilibrium is at E_2, the impact effect is at A$'$, and the saddle path is the dashed line from A$'$ to E_2. The transition path of the capital stock and its associated shadow value is qualitatively similar to that under lump-sum tax recycling, with the capital stock decreasing gradually to its new steady-state value and Tobin's q returning to its old equilibrium value upon an initial fall. Not surprisingly, the qualitative allocation effects under labor tax recycling are essentially the same as those under lump-sum tax recycling, except for the short-run employment effect. The adjustment speed to the new equilibrium is lower, however, and the quantitative allocation effects are different too. All quantitative effects are less pronounced under labor tax recycling because the fall in the capital stock that generates all dynamics is smaller in the labor tax reform scenario. Table 3 summarizes the effects on the main variables and compares them with the lump-sum tax recycling scenario.

On impact, the shadow value of the capital stock falls, though by less than under lump-sum recycling, reflecting a compensating labor supply response. This ensures that the path of the after-tax marginal product of capital is positioned above the corresponding path for lump-sum tax recycling, both on impact and during transition.[26] Employment rises at impact due to the labor tax cut, which shifts the labor supply curve down, thereby lowering short-run wages. The relative size of the impact effect on full income under the two scenarios is ambiguous. It can be shown that $[\tilde{Y}_F(0)]_{LS} = [\tilde{Y}_F(0)]_{LT}$ if labor supply is exogenous (that is, $\theta = 0$), where 'LT' refers to labor tax recycling. But, $[\tilde{Y}_F(0)]_{LS} > [\tilde{Y}_F(0)]_{LT} > 0$ if labor supply is endogenous (that is, $\theta > 0$) provided the initial labor tax rate is low.[27] Full income rises at impact due to the labor tax cut, but the increase is smaller than under lump-sum tax recycling. The reason is that part of the labor tax cut is passed on to capital owners in the form of lower wages. If initial labor tax rates are high, however, the degree of tax shifting is lower and the positive effect on the after-tax wage rate will start to dominate the rise in transfers, so that $\tilde{Y}_F(0)]_{LT} > [\tilde{Y}_F(0)]_{LS}$.

The fall in the long-run capital stock is smaller if households enjoy a cut in labor taxes rather than an increase in lump-sum transfers. The increase in labor supply induced by the fall in the labor tax dampens the reduction in the capital stock required to keep the after-tax rate of return on capital at its equilibrium value. Accordingly, if labor supply is exogenous, the two scenarios are equivalent, that is, $[\tilde{K}(\infty)]_{LT} = [\tilde{K}(\infty)]_{LS} = -\tilde{\tau}_K/\varepsilon < 0$. Full income falls in the long run for the same reasons as under lump-sum tax recycling, but the reduction

[26] Since Tobin's q represents the present value of this path, its path under tax reform also lies above the lump-sum path. See Heijdra, Kooiman and Ligthart (2004).

[27] A necessary and sufficient condition for this result is that $\tau_L < (1-\varepsilon)(1-\tau_K)$. See Heijdra, Kooiman and Ligthart (2004).

is smaller. Compensating labor market adjustments ensure that steady-state labor earnings exceed the level attained under lump-sum revenue recycling. Evidently, if labor supply is exogenous the two scenarios coincide, just like if initial taxes are zero, in which case full income is unaffected.

Is there an employment double dividend? The process of capital decumulation set in motion by the rise in environmental taxes improves environmental quality without immediately offsetting the positive employment effect. Thus, in the short run, an employment double dividend is obtained. The environmental gain is persistent and growing, but the employment dividend is eroded during transition. Intuitively, the decrease in the physical capital stock depresses labor demand. The concomitant erosion of the tax base necessitates a gradual increase in the labor tax rate, which, in turn, reduces labor supply. Both effects result in a lower level of aggregate employment. At best, if $\tau_K = 0$ initially, the long-run employment effect is zero. Of course, if the short-run Laffer-curve condition (SRL) is violated, stability then requires SC in Assumption 2 to switch sign, so that employment rises in the long run. At the same time, the steady-state capital stock falls, thereby sustaining a green dividend. Though theoretically possible, this scenario has little practical value because labor supply is unlikely to be elastic enough to violate the SRL condition, except for unrealistically high initial tax rates.

5.3 Distributional Effects

Now that the dynamic allocation effects of the green tax reform have been determined, we can follow a similar methodology as in Section 4.2 to compute its welfare consequences. It is shown that the welfare changes across generations are qualitatively similar to those under lump-sum tax recycling, but the path of welfare change is more flattened out under labor tax recycling.

The change in financial wealth follows readily from $[\tilde{A}(0)]_{LT} = \omega_A[\tilde{q}(0)]_{LT} < 0$. Using (18), this gives us the change in private welfare of very old generations under labor tax recycling, $[dU_P(-\infty, 0)]_{LT}$. Because the fall in Tobin's q is smaller under labor tax recycling, it immediately follows that old generations experience a smaller welfare loss:

$$[dU_P(-\infty, 0)]_{LS} < [dU_P(-\infty, 0)]_{LT} < 0. \tag{33}$$

The welfare effect for the generations born in the new steady state, $[dU_P(\infty, \infty)]_{LT}$, is derived in a similar manner as in Section 4.2—using that full income falls by less under labor tax recycling—so that those generations are hurt too, but by less than under lump-sum tax recycling:

$$[dU_P(\infty, \infty)]_{LS} < [dU_P(\infty, \infty)]_{LT} \leq 0. \tag{34}$$

Using the second expression of (18) and (T2.8a), it immediately follows that generations born at the time of the implementation of the tax reform benefit, but we cannot readily compare this with the gain under lump-sum tax recycling. This problem exists because the impact effect on human wealth is a weighted average of the impact and long-run effects on full income, where both the weights differ ($h_I > h_I^*$) and the quantities to be weighted differ (that is, $[\tilde{Y}_F(0)]_{LT} < [\tilde{Y}_F(0)]_{LS}$ for low tax rates and $[\tilde{Y}_F(\infty)]_{LT} > [\tilde{Y}_F(\infty)]_{LS}$ otherwise). No general conclusions can be drawn, but some special cases are nevertheless instructive. First, if capital is highly mobile ($\sigma \to 0$) then both h_I and h_I^* are large relative to λ, and the long-run effect on full income dominates, so that $[\tilde{H}(0)]_{LT} > [\tilde{H}(0)]_{LS}$. Second, if the generational-turnover effect is strong (λ is high), the short-run effect on full income dominates so that $[\tilde{H}(0)]_{LT} < [\tilde{H}(0)]_{LS}$ for low tax rates. The last inequality also holds generally for zero initial taxes and thus for initial taxes close to zero.

Substituting the change in the long-run capital stock (see $[\tilde{K}(\infty)]_{LT}$ in Table 3) into (21), and noting that the adjustment speed is h_I^*, we can conclude that the rise in environmental welfare is smaller under labor tax rebating:

$$[dU_E(0)]_{LS} > [dU_E(0)]_{LT} = -\left(\frac{h_I^*}{\rho+\lambda+h_I^*}\right)\left(\frac{\alpha_E}{\rho+\lambda+\alpha_E}\right)\frac{\alpha_K\left[\tilde{K}(\infty)\right]_{LT}}{\rho+\lambda} > 0. \quad (35)$$

The smaller drop in the capital stock and the slower adjustment both cause a smaller rise in environmental welfare in the short run. In the long run, only the capital stock effect survives (see (22), which does not feature h_I), ensuring that $[dU_E(\infty)]_{LS} > [dU_E(\infty)]_{LT}$. Proposition 4 shows that the qualitative effects are similar to those of Proposition 2, although the required conditions are not necessarily similar.

Proposition 4 *A rise in environmental taxes of which the revenues are used to cut labor taxes in a revenue-neutral fashion yields the following changes in welfare across generations:*
(i) $[dU_P(-\infty,0)]_{LT} < 0$;
(i) $[dU_P(0,0)]_{LT} > 0$ *for low initial tax rates;*
(iii) $[dU_P(\infty,\infty)]_{LT} < 0$;
(iv) $[dU_P(-\infty,0)]_{LT} < [dU_P(0,0)]_{LT}$ *for low initial tax rates;*
(v) $[dU_P(\infty,\infty)]_{LT} < [dU_P(0,0)]_{LT}$ *for all initial tax rates; and*
(vi) $0 < [dU_E(0)]_{LT} < [dU_E(\infty)]_{LT}$ *for all initial tax rates.*

PROOF: *See Heijdra, Kooiman and Ligthart (2004).*

The qualitative welfare results are unaffected by the method of rebating environmental tax revenue. Of course, for specific generations the consequences of the two policies may differ, but the broad picture is by and large the same. The main differences between the two policies are that labor tax recycling: (i) generates slower adjustment to the new steady state and consequently affects more generations; and (ii) yields quantitative allocation effects that are generally smaller. Welfare changes under labor tax rebating are smaller in 'amplitude' but greater in 'spread' (that is, the number of generations being affected). As a result, old generations and generations born in the new steady state enjoy a smaller welfare loss, while generations born at the time of the policy change gain less. We cannot unambiguously rank one of the two policies as being preferred. A smaller amplitude is socially desirable, but a greater spread is not since it generates a larger need for intergenerational redistribution. A policy maker having to choose between the two policies faces a trade-off between, on the one hand, more generations being affected, and, on the other hand, the impact being smaller. Corollary 1 specifies the differences in welfare effects of the two revenue-recycling policies:

Corollary 1 *Comparing the welfare changes of a rise in environmental taxes under lump-sum tax recycling and labor tax recycling yields:*
(i) $[dU_P(-\infty,0)]_{LS} < [dU_P(-\infty,0)]_{LT}$ *for all feasible rates of initial taxes;*
(ii) $[dU_P(0,0)]_{LS} > [dU_P(0,0)]_{LT}$ *for low initial tax rates;*
(iii) $[dU_P(\infty,\infty)]_{LS} < [dU_P(\infty,\infty)]_{LT}$ *for all feasible rates of initial taxes;*
(iv) $[dU_E(0)]_{LS} > [dU_E(0)]_{LT}$; *and*
(v) $[dU_E(\infty)]_{LS} > [dU_E(\infty)]_{LT}$ *for all feasible initial tax rates.*

6 Concluding Remarks

The paper has explored the dynamic allocation effects and intergenerational welfare implications of a balanced budget rise in environmental taxes. Consider first the case where tax revenues are returned to households in a lump-sum fashion. Environmental taxes are shown to have negative effects on physical capital accumulation and aggregate employment in the new steady state, thereby improving long-run environmental quality. Since capital is the only pollution-producing factor, it is optimal to tax capital income directly, despite that labor is ultimately bearing the incidence of the environmental tax.

By internalizing pollution externalities, environmental taxes enhance economic efficiency, but yield uneven effects on the intergenerational distribution of income and welfare. In particular, old generations suffer losses in private welfare, owing to a fall in capital income, whereas young generations—primarily consuming out of human capital—experience smaller welfare losses. In fact, their private welfare may even rise if they are born close to the time of implementation of the policy change and initial taxes are not too high. Future generations enjoy a rise in environmental welfare from a larger stock of natural capital, but suffer a private welfare loss on account of the smaller stock of physical capital they have to work with. Generations born further into the future experience the largest environmental welfare gain, because they are born in the cleanest world. Accordingly, without further supporting transfers, environmental taxes may be political hard to implement as current generations are the ones that vote on new policies.

Policy makers can employ bond policy to neutralize the uneven welfare effects of environmental taxes. By initially running a fiscal deficit, financed by issuing bonds, the government can provide lump-sum transfers to existing generations to compensate for their private welfare losses. Generations born after the policy change—which are living in the cleanest world, and thus are better off in terms of environmental welfare—bear the burden of debt redemption. In that way, all generations enjoy the same welfare gain from environmental taxes.

Rebating environmental tax revenue in the form of a cut in labor income taxes—assuming the economy operates on the upward-sloping segment of the Laffer curve—yields a rise in employment in the short run, reflecting the positive labor supply response induced by the fall in labor taxes. In the long run, employment falls, however, but by less than under lump-sum tax recycling, implying that the employment double dividend is a short-run phenomenon in an open economy. Compared to lump-sum rebating the distribution of welfare changes across generations is flattened out more. Accordingly, old existing generations and future generations lose out less, whereas generations born at the time of the shock gain less.

The paper could be usefully extended in a number of directions. First, environmental quality was modeled to enter the household's utility function in a separable fashion, though widely accepted as a convenient simplification, it is rather restrictive. More general specifications of the utility function can take into account interaction effects: a cleaner environment may encourage households to consume more leisure. Furthermore, the interaction between the environment and the production side of the economy could be fleshed out more. Including the renewable resource as an input into production is a fruitful extension.[28]

Appendix

A.1 Log-linearized Model

To solve the model we log-linearize the equations of Table 1 (in the main text) around an initial steady state, where $B = F = 0$ holds initially. The linearization is based on the standard definition, that is, $\tilde{x}(t) \equiv dx(t)/x$ for most of the variables, except for: (i) time derivatives, which are scaled by the steady-state level of a variable rather than the steady-state change since the latter is generally zero, for example, $\dot{\tilde{x}}(t) \equiv \dot{x}(t)/x$; (ii) financial assets and human capital (that is, $A(t)$, $B(t)$, $F(t)$, and $H(t)$), which are scaled by output and multiplied by r, for example, $\tilde{A}(t) \equiv rdA(t)/Y$, and $Y_F(t)$, $T(t)$ and $Z(t)$, which are scaled by output, for example, $\tilde{Y}_F(t) \equiv dY_F(t)/Y$; and (iii) income tax rates, which are defined as $\tilde{\tau}_j(t) \equiv d\tau_j(t)/(1-\tau_j)$, where $j = \{K, L\}$. We further use shares ω_C, ω_X, ω_I and ω_A which are all defined as the ratio of the respective variable over net output, except for the share of financial wealth which is pre-multiplied by r. The results of the linearization are reported in the Appendix Table.

By using (AT.6), (AT.7), and (AT.9), we obtain 'quasi-reduced form' expressions for employment, $\tilde{L}(t)$, the wage rate, $\tilde{w}(t)$, and aggregate net output, $\tilde{Y}(t)$ as functions of the

[28] Mourmouras (1993) includes natural resources as an input to production in a Diamond OLG model. He studies optimal environmental taxes, however.

capital stock:

$$\tilde{L}(t) = \frac{\theta\left[(1-\varepsilon)\tilde{K}(t) - \tilde{\tau}_L(t)\right]}{1+\theta(1-\varepsilon)}, \tag{A.1}$$

$$\tilde{w}(t) = \frac{(1-\varepsilon)\tilde{K}(t) + \theta(1-\varepsilon)\tilde{\tau}_L(t)}{1+\theta(1-\varepsilon)}, \tag{A.2}$$

$$\tilde{Y}(t) = \frac{(1+\theta)(1-\varepsilon)\tilde{K}(t) - \varepsilon\theta\tilde{\tau}_L(t)}{1+\theta(1-\varepsilon)}. \tag{A.3}$$

A.2 Environmental Taxes Under Lump-Sum Tax Recycling

The stability of the investment system is investigated by using (AT.15) and (AT.8) in (AT.3)-(AT.4) and writing the resulting expressions in matrix format:

$$\begin{aligned}\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{q}}(t) \end{bmatrix} &= \begin{bmatrix} 0 & \frac{r\omega_I}{\sigma\omega_A} \\ \frac{r\varepsilon(\omega_A+\omega_I)}{\omega_A[1+\theta(1-\varepsilon)]} & r \end{bmatrix} \begin{bmatrix} \tilde{K}(t) \\ \tilde{q}(t) \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ \left(\frac{r(\omega_A+\omega_I)}{\omega_A}\right)\left[\tilde{\tau}_K(t) + \left(\frac{\theta\varepsilon}{1+\theta(1-\varepsilon)}\right)\tilde{\tau}_L(t)\right] \end{bmatrix},\end{aligned} \tag{A.4}$$

where the Jacobian matrix on the right-hand side has the characteristic polynomial:

$$p_I(x) = x(r-x) + r\left(\frac{r\omega_I}{\sigma\omega_A}\right)\left(\frac{\omega_A+\omega_I}{\omega_A}\right)\left(\frac{\varepsilon}{1+(1-\varepsilon)\theta}\right), \tag{A.5}$$

which has distinct roots $-h_I < 0$ and $r_I = r + h_I > r$. This shows that the investment system is stable if labor income taxes are exogenously given, as in Sections 3 and 4 of the main text. By noting $p_I(-h_I) = 0$ and implicitly differentiating (A.5) with respect to h_I and σ we obtain:

$$-(r+2h_I)\frac{\partial h_I}{\partial\sigma} = \frac{p_I(0)}{\sigma} > 0,$$

that is, the adjustment speed is negatively related to σ.

In a similar fashion, the stability of the savings system can be shown by writing (AT.1)-(AT.2) in the matrix form:

$$\begin{bmatrix} \dot{\tilde{H}}(t) \\ \dot{\tilde{A}}(t) \end{bmatrix} = \begin{bmatrix} r+\lambda & 0 \\ -(\rho+\lambda) & r-(\rho+\lambda) \end{bmatrix}\begin{bmatrix} \tilde{H}(t) \\ \tilde{A}(t) \end{bmatrix} - \begin{bmatrix} r\tilde{Y}_F(t) \\ -r\tilde{Y}_F(t) \end{bmatrix}, \tag{A.6}$$

where $\tilde{Y}_F(t)$ is the time-varying shock term of the savings system, that is, the shock in full income given in (AT.11). The savings system has the following characteristic polynomial:

$$p_S(x) = (r-(\rho+\lambda)-x)(r+\lambda-x),$$

which has distinct roots $-h_S = r-(\rho+\lambda) < 0$ and $r_S = r+\lambda > 0$. Hence, the savings system is also stable. Proposition 1 in the main text summarizes the results.

Equation system (A.4) can be solved in steady state to arrive at expressions for $\tilde{K}(\infty)$ and $\tilde{q}(\infty)$. Using (A.1)-(A.3) and the equations in the Appendix Table, expressions for the other variables can be derived. Table 2 in the main text summarizes the results. As further explained in Heijdra, Kooiman and Ligthart (2004), we can now also derive transition paths. For example, the path for full consumption is a weighted average of the impact and the steady-state effect:

$$\tilde{Y}_F(t) = \tilde{Y}_F(0)e^{-h_I t} + \left[1 - e^{-h_I t}\right] \tilde{Y}_F(\infty). \tag{A.7}$$

A.3 Environmental Taxes Under Labor Tax Recycling

Under labor tax recycling, (31), (AT.3), (AT.4), and (AT.8) are combined to obtain the matrix expression for the investment system:

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{q}}(t) \end{bmatrix} &= \begin{bmatrix} 0 & \frac{r\omega_I}{\sigma\omega_A} \\ \frac{r(\omega_A+\omega_I)\psi(\tau_K,\tau_L)}{\omega_A} & r \end{bmatrix} \begin{bmatrix} \tilde{K}(t) \\ \tilde{q}(t) \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ \left(\frac{r(\omega_A+\omega_I)}{\omega_A}\right) \chi(\tau_K,\tau_L)\tilde{\tau}_K(t) \end{bmatrix}, \end{aligned} \tag{A.8}$$

where $\psi(\tau_L, \tau_K)$ and $\chi(\tau_L, \tau_K)$ are given in (32) in the main text. Stability holds provided the determinant of the Jacobian matrix on the right-hand side is negative, that is $\psi(\tau_L, \tau_K) > 0$. The SRL condition in Assumption 1 ensures that the denominator of $\psi(\tau_L, \tau_K)$ is positive, so that the stability condition is met provided Assumption 2 holds (and thus the numerator of $\psi(\tau_L, \tau_K)$ is positive also).

The inequality $h_I^* < h_I$ is proved by noting that the characteristic polynomial associated with the Jacobian matrix on the right-hand side of (A.7) is:

$$p_I^*(x) = p_I(x) - r\left(\frac{r\omega_I}{\sigma\omega_A}\right)\left(\frac{\omega_A + \omega_I}{\omega_A}\right)\left[\frac{\varepsilon}{1 + (1-\varepsilon)\theta} - \psi(\tau_K, \tau_L)\right]. \tag{A.9}$$

It is straightforward to show that, provided $\theta\omega_Z > 0$, $p_I^*(x)$ is a vertical shift down from $p_I(x)$, that is, the term in square brackets on the right-hand side of (A.9) is positive. This proves that $r_I^* < r_I$ and $h_I^* < h_I$.

As the shock term of the savings system, $\tilde{Y}_F(t)$, is time varying, standard comparative static analysis does not deliver a solution for the transition. Instead, we apply the Laplace transform method pioneered by Judd (1982, 1985) and developed by, among others, Bovenberg (1993).[29]

[29] The Laplace transform is defined as:

$$\mathcal{L}\{x, s\} \equiv \int_0^\infty x(t)e^{-st}dt,$$

which can be interpreted as the net present value of the path of $x(t)$ discounted at rate s.

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Appendix Table. Log-Linearized Model

$$\dot{\tilde{A}}(t) = [r - (\rho + \lambda)]\, \tilde{A}(t) - (\rho + \lambda)\tilde{H}(t) + r\tilde{Y}_F(t) \quad \text{(AT.1)}$$

$$\dot{\tilde{H}}(t) = (r + \lambda)\tilde{H}(t) - r\tilde{Y}_F(t) \quad \text{(AT.2)}$$

$$\dot{\tilde{K}}(t) = (r\omega_I/\omega_A)\left[\tilde{I}(t) - \tilde{K}(t)\right] \quad \text{(AT.3)}$$

$$\dot{\tilde{q}}(t) = r\tilde{q}(t) - [r(\omega_A + \omega_I)/\omega_A]\left[\tilde{Y}(t) - \tilde{K}(t) - \tilde{\tau}_K(t)\right] \quad \text{(AT.4)}$$

$$\dot{\tilde{E}}(t) = -\alpha_E\left[\tilde{E}(t) + \alpha_K\tilde{K}(t)\right] \quad \text{(AT.5)}$$

$$\tilde{w}(t) = \tilde{Y}(t) - \tilde{L}(t) \quad \text{(AT.6)}$$

$$\tilde{L}(t) = \theta\,[\tilde{w}(t) - \tilde{\tau}_L(t)] \quad \text{(AT.7)}$$

$$\tilde{q}(t) = \sigma\left[\tilde{I}(t) - \tilde{K}(t)\right] \quad \text{(AT.8)}$$

$$\tilde{Y}(t) = \varepsilon\tilde{L}(t) + (1-\varepsilon)\tilde{K}(t) \quad \text{(AT.9)}$$

$$\tilde{T}(t) = [(1-\varepsilon)\tau_K + \varepsilon\tau_L]\,\tilde{Y}(t) + (1-\varepsilon)(1-\tau_K)\tilde{\tau}_K(t) + \varepsilon(1-\tau_L)\tilde{\tau}_L(t) \quad \text{(AT.10)}$$

$$\tilde{Y}_F(t) = \varepsilon(1-\tau_L)\,[\tilde{w}(t) - \tilde{\tau}_L(t)] + \tilde{Z}(t) \quad \text{(AT.11)}$$

$$\omega_C\tilde{C}(t) = \omega_X\tilde{X}(t) + \theta\left[\tilde{Y}_F(t) - \tilde{Z}(t)\right] \quad \text{(AT.12)}$$

$$\omega_X\tilde{X}(t) = ((\rho + \lambda)/r)\left[\tilde{A}(t) + \tilde{H}(t)\right] \quad \text{(AT.13)}$$

$$\tilde{A}(t) = \omega_A\left[\tilde{K}(t) + \tilde{q}(t)\right] + \tilde{B}(t) + \tilde{F}(t) \quad \text{(AT.14)}$$

$$\dot{\tilde{B}}(t) = r\left[\tilde{B}(t) + \tilde{Z}(t) - \tilde{T}(t)\right] \quad \text{(AT.15)}$$

Initial shares:
$\omega_I \equiv I/Y$, share of firm investment in national income; $\omega_X \equiv X/Y$, share of full consumption in national income; $\omega_A \equiv rA/Y = rqK/Y$, share of asset income in national income; $\omega_C \equiv C/Y$, share of consumption in national income; $\omega_Z \equiv Z/Y = T/Y$, share of transfers and tax revenue in national income.

Relationships between initial shares and parameters:
$\omega_C = \omega_A + (1-\tau_L)\varepsilon$, $\omega_C = 1 - \omega_I$, $\omega_C = \omega_X + \theta\varepsilon(1-\tau_L)/(1+\theta)$, $\omega_A + \omega_I = (1-\varepsilon)(1-\tau_K)$, $\omega_Z = \varepsilon\tau_L + (1-\varepsilon)\tau_K$

Notes:
(a) We have used the normalization $B = F = 0$ initially; (b) $\sigma \equiv -(I/K)(\phi''/\phi') \geq 0$; and (c) $\alpha_E \equiv -f_E > 0$ and $\alpha_K \equiv -(f_K/f_E)(K/E) > 0$.

Table 2: Allocation Effects of Environmental Taxes Under Lump-Sum Recycling

Variable	Impact Effect		Steady-State Effect	
$\tilde{K}(t)$	0	(T2.1a)	$-\frac{1+\theta(1-\varepsilon)}{\varepsilon} < 0$	(T2.1b)
$\tilde{L}(t)$	0	(T2.2a)	$-\frac{\theta(1-\varepsilon)}{\varepsilon} < 0$	(T2.2b)
$\tilde{q}(t)$	$-\left(\frac{(1-\varepsilon)(1-\tau_K)}{\omega_A}\right)\left(\frac{r}{r+h_I}\right) < 0$	(T2.3a)	0	(T2.3b)
$\tilde{w}(t)$	0	(T2.4a)	$-\left(\frac{1-\varepsilon}{\varepsilon}\right) < 0$	(T2.4b)
$\tilde{Y}_F(t)$	$(1-\varepsilon)(1-\tau_K) > 0$	(T2.5a)	$-(1-\varepsilon)\left[\theta(\tau_L-\tau_K)+\left(\frac{1+\theta}{\varepsilon}\right)\tau_K\right] \leq 0$	(T2.5b)
$\tilde{Y}(t)$	0	(T2.6a)	$-\frac{(1-\varepsilon)(1+\theta)}{\varepsilon} < 0$	(T2.6b)
$\tilde{A}(t)$	$\omega_A\tilde{q}(0) < 0$	(T2.7a)	$\left(\frac{r}{r+\lambda}\right)\left(\frac{r-\rho}{h_S}\right)\tilde{Y}_F(\infty) \leq 0$	(T2.7b)
$\tilde{H}(t)$	$\left(\frac{r}{r+\lambda}\right)\left[\left(\frac{r+\lambda}{r+\lambda+h_I}\right)\tilde{Y}_F(0)+\left(\frac{h_I}{r+\lambda+h_I}\right)\tilde{Y}_F(\infty)\right]$	(T2.8a)	$\left(\frac{r}{r+\lambda}\right)\tilde{Y}_F(\infty) \leq 0$	(T2.8b)
$\tilde{E}(t)$	0	(T2.9a)	$-\alpha_K\tilde{K}(\infty) > 0$	(T2.9b)

Table 3: Summary of the Effects of Labor Tax Rebating Compared With Lump-Sum Rebating

Impact Effect	Steady-State Effect
$[\tilde{K}(0)]_{LS} = [\tilde{K}(0)]_{LT} = 0$	$[\tilde{K}(\infty)]_{LS} < [\tilde{K}(\infty)]_{LT} = -\frac{1-(1+\theta)\tau_L}{\varepsilon[1-(1+\theta)\tau_L]-\theta(1-\epsilon)\tau_K} < 0$
$[\tilde{L}(0)]_{LT} = \frac{\theta(1-\varepsilon)(1-\tau_K)}{\varepsilon[1+\theta(1-\varepsilon)+\theta(1-\varepsilon)(1-\tau_K)]} > [\tilde{L}(0)]_{LS} = 0$	$[\tilde{L}(\infty)]_{LS} < [\tilde{L}(\infty)]_{LT} = -\frac{\theta(1-\varepsilon)\tau_K}{\varepsilon[\varepsilon[1-\tau_L(1+\theta)]-\theta(1-\varepsilon)\tau_K]} \leq 0$
$[\tilde{q}(0)]_{LS} < [\tilde{q}(0)]_{LT} = -\left(\frac{(1-\varepsilon)(1-\tau_K)\chi(\tau_K,\tau_L)}{\omega_A}\right)\left(\frac{r}{r+h_I^*}\right) < 0$	$[\tilde{q}(\infty)]_{LS} = [\tilde{q}(\infty)]_{LT} = 0$
$0 < [\tilde{Y}_F(0)]_{LT} = \left(\frac{1-\tau_L}{1-(1+\theta)\tau_L+\theta(1-\varepsilon)(1-\tau_K)}\right)\left[\tilde{Y}_F(0)\right]_{LS}$	$[\tilde{Y}_F(\infty)]_{LS} \leq [\tilde{Y}_F(\infty)]_{LT} = -\frac{(1-\varepsilon)(1-\tau_L)\tau_K}{\varepsilon[1-(1+\theta)\tau_L]-\theta(1-\varepsilon)\tau_K} \leq 0$